How Tall is That Building?

Objectives:

- .To find the height of a building.

Learning Outcomes:

I can measure accurately with a ruler.

I can work with metres and centimetres and convert between them.

I can carry out calculations using decimals, with and without a calculator.

I understand similar triangles.

I can use ratio.

Method: A series of lessons using a combination of whole-class teaching, pair, group and individual work.

What you need: A building made of visible bricks. Metre sticks. A sunny day.

Before the Lesson

Identify a brick-built building where the bricks are visible. Ideally choose one with windows, and/or which has a drainpipe in sections, and set the challenge on a sunny day when it is possible to see the length of the shadow of the main building wall. Take care that the shadow is not likely to get tangled with the shadows of any trees, and make sure that it is clear where the shadow of the main wall ends and that of the roof begins. (Roofs with eaves may present a problem.)

Introduction

Explain to the pupils that they are going to carry out a measuring challenge outdoors. Revise any previous teaching on potential sources of error when measuring and methods for dealing with them, eg.

- Involve two team members in every measurement, one measuring and the other watching.
- Make sure that measurements begin from exactly the right place.
- Draw diagrams and record measurements carefully, indicating clearly what they are.
- Repeat measurements several times and take an average.

Explain to the pupils that you are going to ask them to write a full account of their investigating, so they will need to keep a full record of their calculations and their thinking.

Divide the pupils into groups and issue each group with a metre stick and a calculator. Take the pupils outside and challenge them to work out the height of the building (excluding the roof) as accurately as they can, using whatever method they wish as long as they remain on the ground! Circulate and discuss the methods as the pupils work. If pupils are completely stuck, the pupil sheet 'Some Ideas' may help.

First Plenary

Gather the pupils together and ask each group to feed back to the class on their method so far and any answer they got. Discuss the advantages and challenges of each method and its potential sources of error. Possible areas of discussion follow.

Counting the Bricks

Several groups are likely to have come up with some version of counting the bricks. With any form of counting, there is clearly the potential of ***losing count***.

To minimise this error, some groups may have tried a straight count and repeated it several times, possibly with different pupils counting. Ask the pupils how many times they think this counting should be done. What kind of average should they use?

- Mean – measure several times, add together and divide by the total number – a bit cumbersome in this case!
- Median – measure several times, arrange the numbers in order and take the middle one – a good option since the correct answer is likely to be near the middle of the different counts.
- Mode – measure several times and take the answer that comes up most often – a good option since they are likely to get the counting correct more often than not.

A good option might be to repeat the counting as often as necessary until the mode and the median give the same answer.

An alternative to straight counting is to use other markers on the building to simplify the task, or to help keep count.

- Use a drainpipe. This only works if the pipe is in sections. Count the number of bricks beside one section and then multiply by the number of sections. Count the extra bricks at the top and bottom of the wall and add these on. Errors may be brought in here if the sections of the pipe are ***not all the same length***.
- Use the windows. Count the number of bricks from the bottom of one window to the bottom of the next and multiply by the number of windows. Count the extra bricks at the top and bottom of the wall and add these on. Errors may be caused here if the windows are ***not all the same height***.

Once the number of bricks has been obtained, we then need a measurement for the thickness of one brick. An obvious error here is to ***forget to measure the mortar*** in between the bricks. A more subtle error is to ***measure only one brick and wedge of mortar*** and assume that this will be the same for all. Any small error in the measurement will be multiplied by the number of bricks and will then become a large error. This can be avoided by taking multiple measurements, but even greater accuracy can be obtained by measuring ten bricks and wedges of mortar and then dividing by ten to get the thickness for one.

Measuring the shadow

Depending on the prior experience of your pupils, some may have thought of measuring the shadow. At a particular moment, the shadow of a simple vertical object will be a particular number of times longer (or shorter) than the height of the object itself. (For example 1.5 times longer.) You can therefore measure a short vertical object like a metre stick or a person, measure its shadow to find this ratio, and then use this in reverse to calculate the height of the building given its shadow length.

Errors here can result from…

- ***inaccuracies in measuring the shadow*** (the shadow of the building may include part of the roof, or there may be an overhang)
- ***inaccuracies in measuring the other object*** (or in ***not holding the object vertically*** when measuring its shadow)
- ***errors in the use of ratio*** (multiplying instead of dividing) etc

Improving Accuracy

Tell the pupils that they are now going to try to improve on their accuracy, either using the same method again, or using one of the other methods. Before they begin, discuss the other potential source of error – inaccurate recording. Ask them to self-evaluate. Have they used diagrams? Is it clear from their notes what each measurement is and what each number refers to? Have they recorded their calculations in full, showing what buttons they pressed on their calculator, or simply typed the numbers in and written down the answer?

Remind the pupils that you are going to ask them to write a full account of their investigation, so they will need to have a full record of their calculations and their thinking.

Writing Up

Back in the classroom, ask the pupils to begin writing up their investigation. (See copymaster 'How tall is that Building?')

Using Shadows and Ratio

Although some pupils may have attempted to use shadows in the foregoing work, it is unlikely that all will be secure with this. Once pupils have had a chance to record sufficient of their initial method that they are able to finish the write-up in their own time (eg for homework) then give them some teaching input as a whole class on ratio. The pupil sheet 'Using Ratio to Calculate Heights' will lead them through the required thinking.

Using Similar Triangles

Depending on the pupils' level of interest, you could also show how to calculate heights using similar triangles. The basic principle is for one pupil to stand looking at the building, while another one holds a metre stick vertically between the building and the first pupil and walks backwards or forwards until the metre stick appears to the observer to be exactly the height of the building. A few measurements and calculations will then establish the actual height.

Show the pupils the pupil sheet 'Using Similar Triangles to Calculate Heights'. Simulate the process in the classroom by trying to find the height of the front wall of the room. Discuss the diagram and put on the appropriate measurements.

Ask the pupils how many times further it is from the observer to the wall, compared with the distance from the observer to the metre stick. Establish, through discussion, that, because of similar triangles, the wall will be this same number of times bigger than the metre stick. Use this thinking to calculate the height of the wall.

Measure the wall from ceiling to floor with metre sticks to see how accurate the method is. Discuss probable sources of error. Repeat the measurements and try to improve on the result.

Send the pupils off in groups to use the same method to calculate the height of your chosen building.

Writing Up

Once the pupils have finished their measuring, remind them of the criteria for writing up their investigation (See copymaster 'How tall is that Building?') Ask them to write-up two different methods and compare the answers.

Name: _____ Class: ____ Date: _____

How Tall is That Building?

Your challenge

To find the height of a building without leaving the ground.

- Decide on your method.
- Consider possible sources of error and decide how to minimise them.
- Keep a careful record of your diagrams, measurements and calculations so that you can write up your investigation later.
- Compare your answer with the answers from other groups. How close are they? Which ones do you think are the most accurate?
- Improve on your accuracy by doing some measuring /calculating again.

Writing Up Your Investigation

- Draw **diagrams.** (clear, labelled, good size, neat)
- Explain **your method**.
- Mention **possible sources of error** and how you minimised them.
- Record your **measurements.**
- Show your **calculations** in full.
- Record your **answer**.

Name: _____ Class: ____ Date: _____

Height of a Building – Some Ideas…

Count the Bricks

We could measure one brick and one strip of mortar.

We could get two people to count and see if they get the same answer.

It would be better to measure ten bricks and divide.

Use the drainpipe

The drainpipe is in sections that are the same length.

There's an extra bit at the top.

We could count bricks.

Use the windows

There are bits in between the windows.

You just need to measure one gap because they are all the same

You could count the bricks in between.

Measure the shadow

The shadow isn't the same as the height of the building.

You could measure me and see how long my shadow is!

We could see how big the shadow of a metre stick is.

Name: _____ Class: ____ Date: _____

Using Ratio to Calculate Heights

Use this method to find the connection between heights and shadow lengths.

1. Start by investigating the length of the shadow of a metre stick held exactly vertically.

Height	**Shadow**
1 m	_____ m

2. Now build this comparison table.

Height	**Shadow**
1 m	_____ m
2 m	_____ m
3 m	_____ m
4 m	_____ m
5 m	_____ m

3. Use your table to answer this question:

How do you **find the length of a shadow** if you **know the height**?

4. Complete:

Height $\xrightarrow{\times \square}$ Shadow

5. So how can you **find the height** if you **know the length of the shadow**?

Height $\xleftarrow{\div \square}$ Shadow

6. The length of shadow for the building is _____

So the height of the building is _____

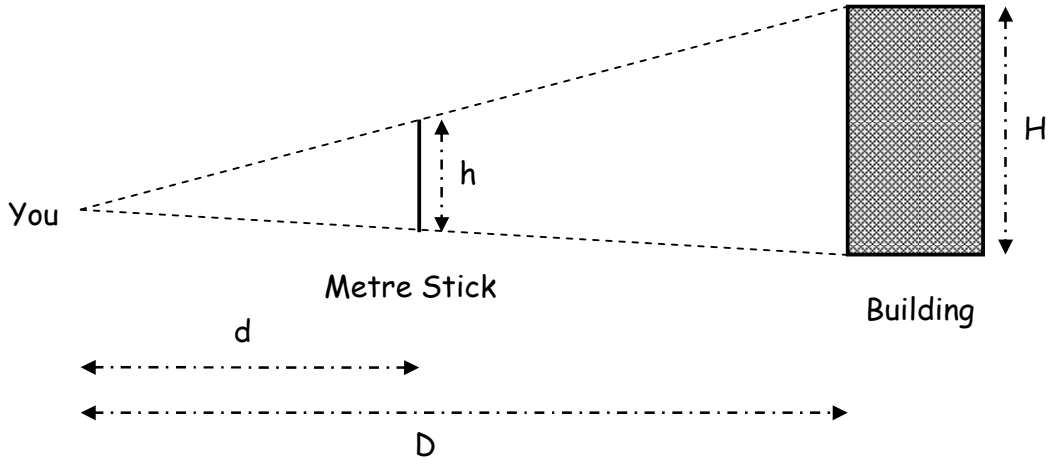
Name: _____ Class: ____ Date: _____

Using Similar Triangles to Calculate Heights

- Stand some distance from the building.

- Your friend stands between you and the building holding a metre stick.

- Ask your friend to move nearer or further away and raise or lower the stick until it appears to you to be exactly the same height as the building.



Carefully measure the distances

- from you to your friend (d)
- from you to the building (D).

Let the height of the metre stick be 'h'.

The unknown height of the building is 'H'.

Use what you know about similar triangles to calculate H.