Formulas for Sequences

Objectives:

- To investigate (linear) sequences and how to work out formulas for them.

Learning Outcomes:

I can make number sequences from various patterns.

I can work out a formula for a number sequence.

I can draw a line graph for a formula.

Method: A series of lessons using a combination of whole-class teaching, pair, group and individual work.

What you need: Pupil worksheets.

Initial Lesson

Show the pupils the two matchstick problems (see copymasters). Set them to investigate, continue each of the patterns and record their answers in suitable tables. If they ask you how far to take the table, suggest they go as far as the tenth pattern.

Finding a Rule

Gather the pupils together and discuss their findings. They should have discovered the following:

1st Problem		**2nd Problem**	
Pattern	**Matchsticks**	**Pattern**	**Matchsticks**
1	4	1	7
2	7	2	12
3	10	3	17
4	13	4	22 etc

Discuss the patterns in the numbers. Establish that in the first problem the number of matchsticks is going up in 3s. In the second problem it is going up in 5s.

The Twentieth Pattern

Give the pupils a couple of minutes to discuss in their groups how many matchsticks they think would be needed for the 'twentieth pattern'.

Share ideas. The likelihood is that you will get a variety of answers. Some pupils may have continued the table till it reaches 20, adding 3 each time. Others may have gone as far as 10 and then doubled it. Some may have tried to work out a formula.

How could we work out who is correct?

How could we accurately work out how many matchsticks are in the hundredth pattern?

Introduce the idea that there might be a magic rule that turns each pattern number into its number of matchsticks. Could we work out what the magic rule is?

Give some examples and try them out.

eg

Add 3 – works on the first pattern in Problem 1 (turns 1 into 4) but doesn't work on the others.

Multiply by 4 – again works on the first pattern in Problem 1 (turns 1 into 4) but doesn't work on the others.

Add 1 and then double it – ditto.

Double it and then add 2 – ditto.

Agree the difference between a 'pattern' and a 'rule'.

- A *pattern* (eg adding 2) is something that allows you to *find one number* in a column *if you know the one above it*.
- A *rule* (or *function*) is something that will change *each* number in the left hand column into its *corresponding number* in the right hand column.

Group Task

Once the pupils have the idea, set them to see if they can find a rule that works for the whole table.

(For your reference (but not the pupils!) the correct answers are:

Problem 1: Multiply by 3 and add 1.

Problem 2: Multiply by 5 and add 2.

Plenary

Gather the pupils together again and see if anyone has managed to discover a rule.

- If they have, that's great! Double check the rule with the whole class to ensure that it is correct.
- If not, that's fine too!

If a rule has been discovered, then ask the pupils whether they can see any connection between the rule and the patterns in the table. (eg in problem 1 the rule tells you to multiply by 3 and the numbers in the table are going in 3s. Discussion: Is this a fluke? Do you think it would always work?)

Suggest that it would be interesting to investigate different rules and their patterns and see if there is any connection between them.

Discuss an appropriate strategy for the investigation. Agree that it would make sense to be **methodical**.

Begin by investigating some 'add' rules and show how they can be written as **formulas**. Eg.

'Add 2' n + 2

then 'Add 4 n + 4

then 'Add 6' n + 6

etc.

Introduce the terminology that a set of numbers generated by a **formula** like this is called a **sequence**. The numbers they in the second column are called the **terms** of the sequence.

Then investigate some 'multiply' rules. Eg.

'Multiply by 3' 3n

then 'Multiply by 4' 4n

then 'Multiply by 5' 5n etc

Then investigate some 'multiply and add' rules. Eg.

'Multiply by 3 and add 1' 3n + 1

then 'Multiply by 3 and add 2' 3n + 2

then 'Multiply by 3 and add 3' 3n + 3 etc

OR

'Multiply by 2 and add 1' 2n + 1

then 'Multiply by 3 and add 1' 3n + 1

then 'Multiply by 4 and add 1' 4n + 1 etc

Group Work

Show the pupils the copymaster 'Investigating Rules' (double-sided) and set them to work. Ask them whether they can discover any connection between each rule and what the pattern is 'going up in'.

Plenary

Gather the pupils together and discuss the patterns that they have found.

First of all look at the multiplying patterns. Ask the pupils if any of them have explored patterns that involve multiplying by 2. Write the tables for these patterns on the board.

Ask what the patterns have in common. Discuss the starting number (different for each pattern.) Discuss what each pattern is going in. (The same for each pattern – they are all going in 2s).

Now explore patterns that involve multiplying by 3. Write the tables for these patterns on the board.

Ask what the patterns have in common. Discuss the starting number (different for each pattern.) Discuss what each pattern is going in. (The same for each pattern – they are all going in 3s).

Hopefully, there will be some excitement at this point!

When the rule has x2 then the numbers go up in 2s. When the rule has x3 then the numbers go up in 3s.

Predict what will happen for other patterns… Get the pupils to check some of their patterns to see if it works.

What would happen if you had a rule that said 'multiply by 1 and add 4?' [This turns out to be the same rule as just 'add 4' since multiplying by 1 leaves a number unchanged.]

What would happen if you had a rule that said 'multiply by 2 and add 0?' [This turns out to be the same rule as just 'multiply by 2' since adding 0 leaves a number unchanged.]

Group Task

Choose one of the sets of patterns – eg the ones that multiply by 3. Ask what is different about them. [They start on different numbers.] Ask if there is any connection between the starting number and the rule.

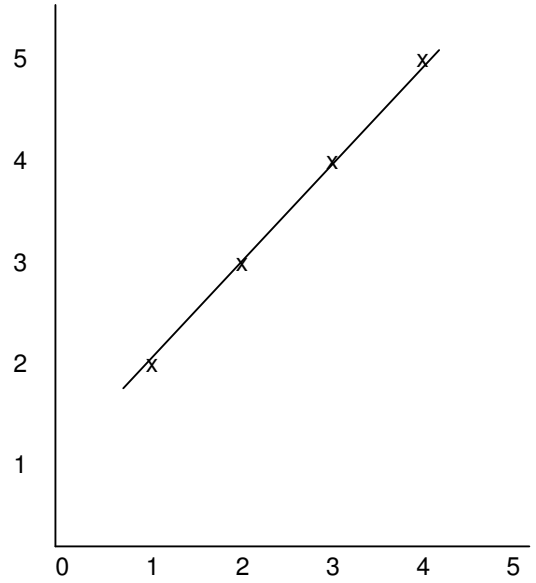
There is not an obvious connection! Accept any suggestion that the pupils come up with and try it out on other patterns. Then set them to investigate in their groups again to see if they can come up with anything consistent.

Plenary

Discuss any findings, but leave the puzzle unsolved for now. All will become plain following the next section of the investigation!

Line Graphs

Introduce the idea of drawing a line graph to show how a pattern behaves.



Introduce the idea of the 'ghost term' (the term before the first term of the sequence). The 'ghost term' is like the 'jumping off point' for the numbers in the sequence.

Get the pupils to draw axes on squared paper and plot the graphs for several related formulas together on the same axes.

Discussion

Can they see any connection between the 'ghost term' and the formula? [The 'ghost term' is the same as the 'add' or 'subtract' number in the formula.]

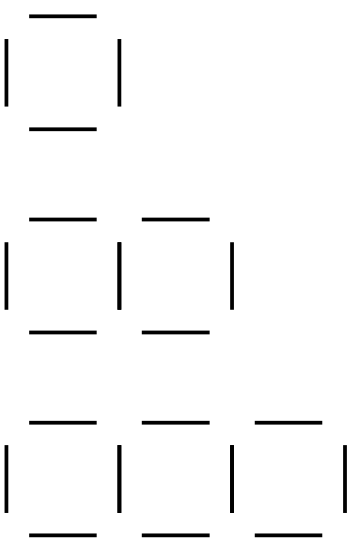
What happens to the line graph if the 'multiply' number in the formula is kept the same and the 'add' or 'subtract' number is altered? [The line moves up or down.] Introduce the terminology that the 'ghost term' – where the line meets the vertical access - is called the **intercept**.

What happens to the line graph if the 'add' or 'subtract' number in the formula is kept the same and the 'multiply' or 'divide' number is altered? [The line becomes steeper or more gentle.] Introduce the terminology that the steepness of the line is called its **gradient**.

Name: _____ Class: ____ Date: _____

The First Matchstick Problem

Matchsticks can be used to make patterns with a row of squares like this;



Pattern 1

Pattern 2

Pattern 3

Pattern	Matchsticks
1	4
2	
3	
4	
5	

Investigate the different number of matchsticks needed to make each pattern.

Continue your pattern for greater numbers of boxes.

Record your discoveries in a table like the one above.

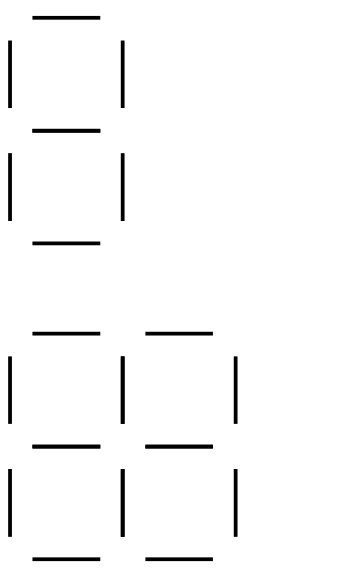
Discuss

- What patterns are there in the numbers?
- Can you work out a rule that will let you find the number of matchsticks for any number of squares?

Name: _____ Class: ____ Date: _____

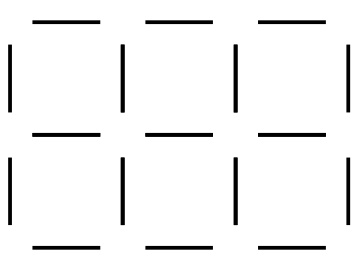
The Second Matchstick Problem

Matchsticks can be used to make patterns with a double row of squares like this;



Pattern 1

Pattern 2



Pattern 3

Pattern	Matchsticks
1	7
2	
3	
4	
5	

Investigate the different number of matchsticks needed to make each pattern.

Continue your pattern for greater numbers of boxes.

Record your discoveries in a table like the one above.

Discuss

- What patterns are there in the numbers?
- Can you work out a rule that will let you find the number of matchsticks for any number of squares?

Name: _____ Class: ____ Date: _____

Investigating Rules

'Add 4'	*'Multiply by 3'*	*'Multiply by 2 and add 1'*
'Add 3'	*'Multiply by -2'*	*'Multiply by 3 and add -2'*

You can use rules like these to make number patterns. Investigate some different rules and see if you can find any connection between the rules and the patterns that you get.

Record the results you get when you apply each rule:

A. Try some 'add' or 'subtract' rules. (eg 'Add 4' or 'Subtract 2'.)

Rule: _____ Rule: _____ Rule: _____

Number	Result
1	
2	
3	
4	
5	

Number	Result
1	
2	
3	
4	
5	

Number	Result
1	
2	
3	
4	
5	

B. Try some 'multiply' or 'divide' rules. (eg 'Multiply by 3' or 'Divide by 2'.)

Rule: _____ Rule: _____ Rule: _____

Number	Result
1	
2	
3	
4	
5	

Number	Result
1	
2	
3	
4	
5	

Number	Result
1	
2	
3	
4	
5	

C. Try some 'two-step' rules. (x/÷ then +/-) (eg ' x by 2 & subtract 4.')

Rule: _____ Rule: _____ Rule: _____

Number	Result
1	
2	
3	
4	
5	

Number	Result
1	
2	
3	
4	
5	

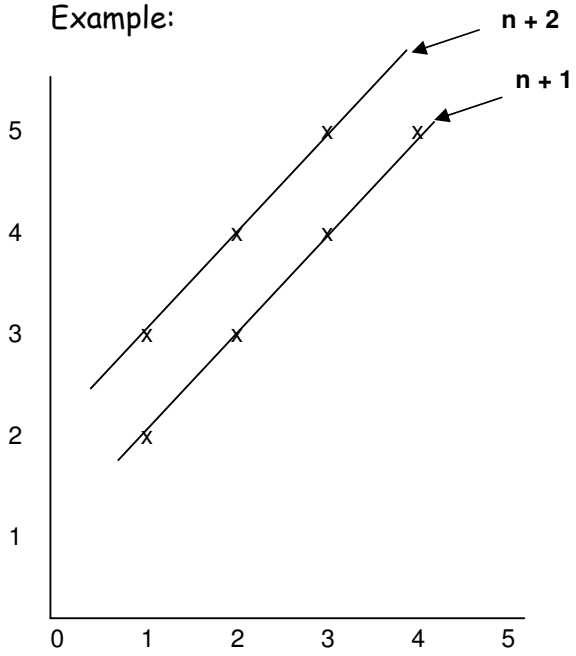
Number	Result
1	
2	
3	
4	
5	

Name: _____ Class: ____ Date: _____

Line Graphs

The effects of several different **formulas** can be compared using line graphs like this.

Example:



n	n + 1	n + 2
0	?	?
1	2	3
2	3	4
3	4	5
4	5	6

What happens if you imagine a 'ghost term' in the sequence *before* the first term? Is there any connection between the 'ghost term' and the formulas?

Draw line graphs for different formulas to investigate.