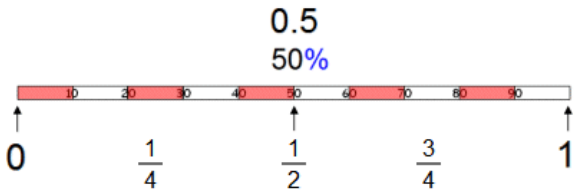
Chopping up a Metre Stick

What's this one all about?



Metre sticks are fabulous things. You can do SO much Maths with them. This investigation gets the children to think about chopping one up into bits!

Chopping it into 2 bits or 4 bits (or 5) is quite straightforward. But what happens when you try to chop it into 3 bits, or 7 bits?

There's **loads** here to investigate – estimating using trial and improvement, the link between multiplication and division and fractions, dealing with remainders, decimals. Set them onto this one and you will keep them happy for hours!

The teacher bits…

Learning Intentions: I can make intelligent estimates and improve on them through trial and improvement. I understand the inverse relationship of multiplication and division. I can interpret remainders in division. I can round calculator answers to an appropriate degree of accuracy. I can find fractions of a quantity.

Age: 8-12

What you need: Class whiteboard, individual whiteboards, pencils & paper, counting sticks, metre sticks.

The investigation

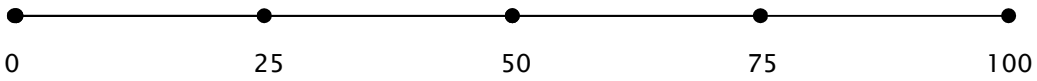
***Note**: The progression below is a guide only. Feel free to depart from it and follow the children's interests. Use your judgement as to how far to take the investigation when you first tackle it.. They can come back and do more next year!*
Numerous photocopy masters are supplied. ***Do not attempt to use them all!*** *You may decide to use individual whiteboards for some parts of the investigation and in other places ask pupils to draw their own tables. Be flexible and have fun!*

Initial Puzzle

Show the pupils a metre stick. Ask where they would need to cut it to chop it in half. What about four equal pieces? What about five? Eight? Ten? Three?

Which of these would be easier? Which harder? Why? (100 divides easily by some numbers, less easily by others.) Establish that this would be an interesting problem to investigate!

Discuss possible ways of recording thinking. Show how a number line could be used:



Show how a table could be used to record all the cutting points for a particular problem.

For example:

Four pieces	
1	25
2	50
3	75
4	100

Set the pupils to work in teams to investigate whichever numbers of pieces you think would be an appropriate level of challenge to begin with.

Development

Once pupils have had a little while to work at the problem, stop them and ask them to share their strategies. If the size of the class permits, this can be done most effectively by inviting other class members to gather around where each group has been working. The following strategies are likely to emerge: (Guess and Check / Trial and Improvement)

- Place markers along the metre stick at equal intervals and count the number of marks between each. Keep adjusting the markers until the intervals are all the same size.

- Estimate the size of one interval and then use repeated addition to count on and see if you reach 100. Adjust and repeat.

Show the pupils how to record their guesses in a table:

1st	2nd	3rd	Result
30	60	90	Too small
35	70	105	Too big
etc			

Once these strategies have been shared, you may wish to allow further time for pupils to work in teams.

Further Development

Ask the *why* question. *Why* does the stick need to be cut at 25 to make 4 pieces?

Why does the stick need to be cut at 20 to make 5 pieces? Establish that 4 x 25 = 100; 5 x 20 = 100.

Establish that the 'easy' problems (eg 4 pieces, 5 pieces, 10 pieces etc) are easy because there is an exact multiplication like this which works. The 'hard' ones (eg 3 pieces) are harder because there isn't.

Explore possible solutions for the '3 pieces' problem. 30 is clearly too small. 35 is too large. 32? 33? 34?

Establish that 3 x 33 = 99. Depending on prior learning, pupils may suggest 33 and a third as the correct answer or they may venture into decimals: 33.3 x 3 = 99.9; 33.33 x 3 = 99.99 etc. If they know about recurring decimals and decimal-fraction equivalence they may note that 1/3 = 0.333 recurring etc. If, on the other hand, they can't solve the problem, leave it unsolved for now!

Lead the discussion to the inverse relationship of multiplication with division. Build patterns as follows:

4 x 25 = 100 100 ÷ 4 = 25 5 x 20 = 100 100 ÷ 5 = 20

Set the pupils to work in teams to see how many of these pairs of facts they can come up with.

At some point in the process pupils will realise that using division will be a way of solving some of the trickier problems. Depending on their prior experience / your objectives you can:

A. **Use written division 'box' sums.** Explore the different ways of recording the answer and the meaning of each of these, vis:

Remainders	Fractions	Decimals
$8\overline{)100}$ = 12r4	$8\overline{)100}$ = 12 ½	$8\overline{)100.0}$ = 12.5

(The remainder is the bit that is left over if you work with whole centimetres. If you want a perfect solution you need to use fractions or decimals to make the pieces '12 and a bit' centimetres long, etc.)

B. Use calculators.

Try dividing 100cm by different numbers. Explore the calculator answers you get. Depending on the pupils' prior experience of decimals (if any) you may need to introduce the idea that eg 16.66666 means 16 'and a bit'. Discuss sensible degrees of accuracy and how to round numbers. If the 'bit' begins with a number that is 5 or more then the 'bit' is more than half so you would round up. If the 'bit' begins with a number that is less than 5 (eg 33.3333) then the 'bit' is less than half so you would round down.

Investigate what happens if you divide 1 (metre) instead of 100cm. Note that the answers to one set of calculations are 100 times bigger than the others. (eg $100 \div 4 = 25$, $1 \div 4 = 0.25$) Discuss the reasons why.

Challenge the pupils to work out (correct to the nearest cm) all the cutting points for dividing the stick into 3 pieces, 6 pieces, 9 pieces, etc. As a starting point, if it has not already come up in discussion, introduce the idea that the pupils have actually been finding 'fractions of a metre'. (one fifth, one sixth etc) Get them to discuss how they would find two fifths if they already know one fifth, how they would find five sixths if they already know one sixth.

Discuss the importance, when working with decimals, of using exact numbers until the calculation is complete. Only round your result when you have reached the final answer. Show how to use a table to record the exact values and the rounded ones.

Thirds	Exact	Rounded
1 third	33.333333cm	33.3cm
2 thirds	66.666666cm	66.7cm
3 thirds	100cm	100cm

Get the children to create their own one-metre lengths of paper and mark on the results of their investigating.

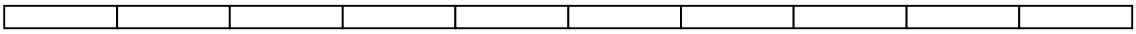
Use the results of their work to discuss equivalent fractions.

Investigate the equivalent decimals for different fractions. (¼ = 0.25 etc). Look for patterns in the numbers. (eg fifths 0.2, 0.4, 0.6 etc)

Name: _____ Class: ____ Date: _____

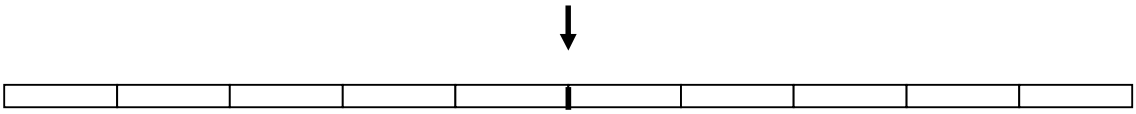
Chopping up a Metre Stick

Here's a diagram of a metre stick.

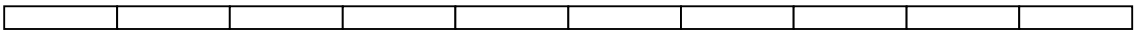


It is 1 metre long!

To cut it in half you would cut it at the 50cm mark.



Where would you cut it if you were to cut it into 4 equal pieces? Why?



What about five equal pieces? Why?

What about ten?

What about three?

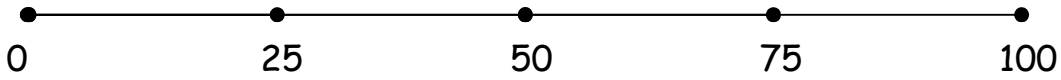
What about six, seven, eight or nine?

Investigate!

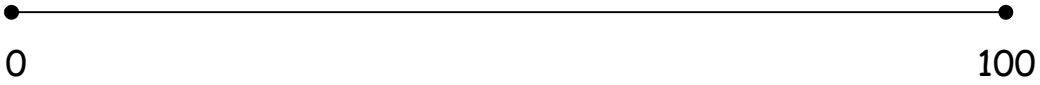
Name: _____ Class: ____ Date: _____

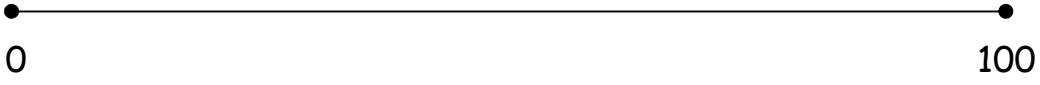
Chopping up a Metre Stick

Here is a number line showing the chopping points for four pieces.



Use these lines to show the chopping points for other numbers of pieces.





Name: _____ Class: ____ Date: _____

Chopping up a Metre Stick

Here's where you would chop a metre stick to make four pieces.

Four pieces	
1	25
2	50
3	75
4	100

Where would you chop it to make five pieces?

Five pieces	
1	
2	
3	
4	
5	

What about ten pieces?

Ten pieces	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

What about eight pieces?

Eight pieces	
1	
2	
3	
4	
5	
6	
7	
8	

Name: _____ Class: ____ Date: _____

Chopping up a Metre Stick

Can you use guess and check to work out where to chop for three pieces?

1st	2nd	3rd	Result
30	60	90	Too small
35	70	105	Too big

What about six pieces?

1st	2nd	3rd	4th	5th	6th	Result
10	20	30	40	50	60	Too small
20	40	60	80	100	120	Too big

Make up your own tables to investigate other numbers of pieces.

Name: _____ Class: ____ Date: _____

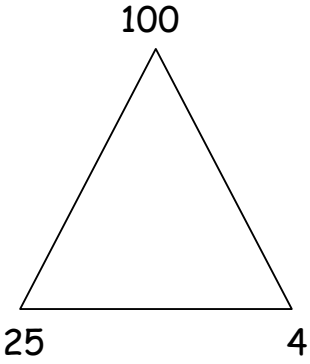
Chopping up a Metre Stick

How many pairs of **whole** numbers can you find that multiply to make 100?

Example: 25 x 4 = 100

25 x 4 = 100

4 x 25 = 100



100 ÷ 4 = 25

100 ÷ 25 = 4

Build these patterns.

1 x 100= 100	100 ÷ 1 = 100
2 x 50 = 100	100 ÷ 2 = 50
x = 100	100 ÷ = ….
x = 100	100 ÷ = ….
x = 100	100 ÷ = ….
x = 100	100 ÷ = ….
x = 100	100 ÷ = ….
x = 100	100 ÷ = ….
x = 100	100 ÷ = ….

What patterns can you see in the patterns?.

Name: _____ Class: ____ Date: _____

Chopping up a Metre Stick

How much of this chart can you complete?

Number of pieces	Length of each piece (cm)
1	100
2	50
3	
4	
5	

Name: _____ Class: ____ Date: _____

Chopping up a Metre Stick

Answers to division calculations can be written in three different ways: with remainders, as fractions, or as decimals.

Make sure you are confident with all of them!

Example: 100 ÷ 8

Twelve eights are 96 (too small)
Thirteen eights are 104 (too big)

100 ÷ 8 = '12 and a bit'

If you count in eights you get 12 eights and four left over.

oooo	oooo	oooo	oooo	oooo	oooo	oooo	oooo	oooo	oooo	oooo	oooo	oo
oooo	oooo	oooo	oooo	oooo	oooo	oooo	oooo	oooo	oooo	oooo	oooo	oo
8	16	24	32	40	48	56	64	72	80	88	96	

100 ÷ 8 = 12r4

The four left over is half of another eight.

100 ÷ 8 = $12\frac{1}{2}$

$\frac{1}{2}$ can be written as 0.5

100 ÷ 8 = 12.5

Name: _____ Class: ____ Date: _____

Chopping up a Metre Stick

Write these answers in three different ways:
with remainders, as fractions, and as decimals.

Example:

$100 \div 8 = 12r4$

$$8\overline{)100}\quad 12r4$$

$100 \div 8 = 12\frac{1}{2}$

$$8\overline{)100}\quad 12\frac{1}{2}$$

$100 \div 8 = 12.5$

$$8\overline{)100.0}\quad 12.5$$

$100 \div 3 =$

$100 \div 3 =$

$100 \div 3 =$

$100 \div 6 =$

$100 \div 6 =$

$100 \div 6 =$

$100 \div 9 =$

$100 \div 9 =$

$100 \div 9 =$

What other ones can you do?

Name: _____ Class: ____ Date: _____

Chopping up a Metre Stick

Work out these divisions with a calculator. Round each answer to the nearest whole number.

Calculation (dividing 100 centimetres)	Calculator answer (cm)	Rounded to nearest whole number (cm)
100 ÷ 2	50	50
100 ÷ 3	33.33333333	33
100 ÷ 4		
100 ÷ 5		
100 ÷ 6		
100 ÷ 7		
100 ÷ 8		
100 ÷ 9		
100 ÷ 10		
100 ÷ 11		
100 ÷ 12		
100 ÷ 13		
100 ÷ 14		
100 ÷ 15		
100 ÷ 16		

What other ones would be interesting to do?

Name: _____ Class: ____ Date: _____

Chopping up a Metre Stick

Work out these divisions with a calculator. Round each answer to two decimal places.

Calculation (dividing 1 metre)	Calculator answer (m)	Rounded to two decimal places (m)
1 ÷ 2	0.5	0.50
1 ÷ 3	0.3333333333	0.33
1 ÷ 4		
1 ÷ 5		
1 ÷ 6		
1 ÷ 7		
1 ÷ 8		
1 ÷ 9		
1 ÷ 10		
1 ÷ 11		
1 ÷ 12		
1 ÷ 13		
1 ÷ 14		
1 ÷ 15		
1 ÷ 16		

What other ones would be interesting to do?

Name: _____ Class: ____ Date: _____

Chopping up a Metre Stick

Carefully stick together four sheets of A4 paper end to end.
Draw a line exactly 1 metre long.

How many fractions can you mark in the correct places?

$\frac{1}{2}$ m?

$\frac{1}{4}$ m?

$\frac{3}{4}$ m?

A fifth of a metre?

Two fifths of a metre?

Four fifths of a metre?

A tenth?

Two tenths?

Nine tenths?

What other ones can you do?

Do any of them come in the same places?

What about *decimal* fractions of a metre?

Name: _____ Class: ____ Date: _____

Chopping up a Metre Stick

Work out these fractions of a metre.

Quarters	
1 quarter	25cm
2 quarters	50cm
3 quarters	75cm
4 quarters	100cm

Fifths	
1 fifth	
2 fifths	
3 fifths	
4 fifths	
5 fifths	

Tenths	
1 tenth	
2 tenths	
3 tenths	
4 tenths	
5 tenths	
6 tenths	
7 tenths	
8 tenths	
9 tenths	
10 tenths	

Eighths	
1 eighth	
2 eighths	
3 eighths	
4 eighths	
5 eighths	
6 eighths	
7 eighths	
8 eighths	

What others can you do?

Name: _____ Class: ____ Date: _____

Chopping up a Metre Stick

Work out these fractions of a metre. First work them out exactly. Then round them to one decimal place.

Thirds	Exact	Rounded
1 third	33.333333cm	33.3cm
2 thirds	66.666666cm	66.7cm
3 thirds	100cm	100cm

Ninths	Exact	Rounded
1 ninth		
2 ninths		
3 ninths		
4 ninths		
5 ninths		
6 ninths		
7 ninths		
8 ninths		
9 ninths		

Sixths	Exact	Rounded
1 sixth		
2 sixths		
3 sixths		
4 sixths		
5 sixths		

Sevenths	Exact	Rounded
1 seventh		
2 sevenths		
3 sevenths		
4 sevenths		
5 sevenths		
6 sevenths		
7 sevenths		

What others can you do?